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# Geometry review, part I

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## Geometry review I

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### Vectors and points

- Geometric vs. coordinate-based approach
- 2D operations on vectors and points
- coordinates, transformations and matrices
- coordinate form of operations

### Lines

- implicit and parametric equations
- intersections, parallel lines

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## Geometry review I

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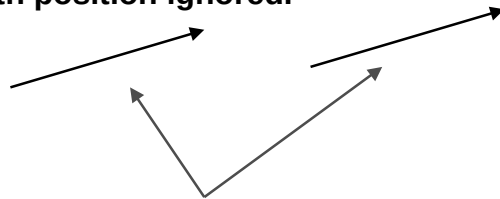
- Vectors and points, 2D
- Lines
- Shapes in the plane
- 3D vectors
- Planes and lines in 3D

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## Geometry vs. coordinates

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Geometric view: a vector is a directed line segment, with position ignored.



different line segments (but with the same length and direction) define the same vector

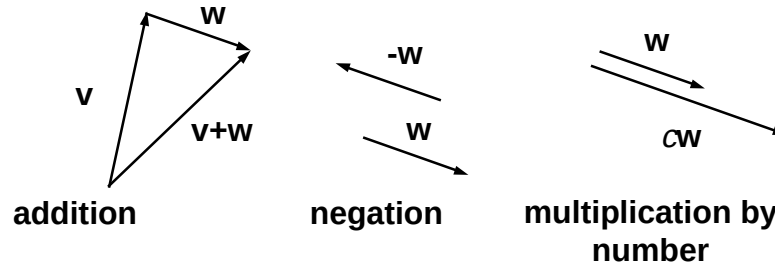
A vector can be thought of as a translation.

Algebraic view: a vector is a pair of numbers

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## Vectors and points

Vector = directed segment with position ignored



Operations on points and vectors:

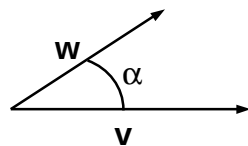
point - point = vector  
point + vector = point

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## Dot product

Dot product: used to compute projections, angles and lengths.

Notation:  $(w,v)$  = dot product of vectors  $w$  and  $v$ .



$$(w,v) = |w| |v| \cos \alpha,$$

$|v|$  = length of  $v$

Properties:

if  $w$  and  $v$  are perpendicular,  $(w,v) = 0$

$$(w,w) = |w|^2$$

angle between  $w$  and  $v$ :  $\cos \alpha = (w,v)/|w||v|$

length of projection of  $w$  on  $v$ :  $(w,v)/|v|$

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## Coordinate systems

For computations, vectors can be described as pairs (2D), triples (3D), ... of numbers.

Coordinate system (2D) =  
point (origin) + 2 basis vectors.

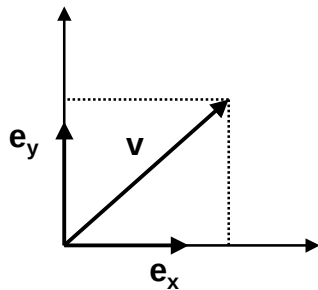
Orthogonal coordinate system:  
basis vectors perpendicular.

Orthonormal coordinate system:  
basis vectors perpendicular and of unit length.

Representation of a vector in a coordinate system:  
2 numbers equal to the lengths (signed) of  
projections on basis vectors.

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## Operations in coordinates



$$\mathbf{v} = \underbrace{(v, e_x)}_{v_x} \mathbf{e}_x + \underbrace{(v, e_y)}_{v_y} \mathbf{e}_y$$

works only for orthonormal  
coordinates!

$$\mathbf{v} = v_x \mathbf{e}_x + v_y \mathbf{e}_y = [v_x, v_y]$$

Operations in coordinate form:

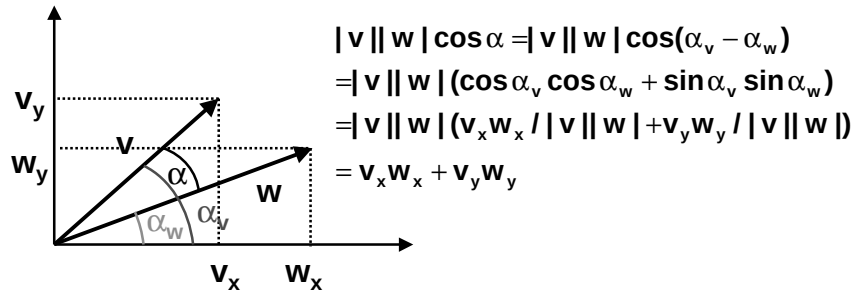
$$\mathbf{v} + \mathbf{w} = [v_x, v_y] + [w_x, w_y] = [v_x + w_x, v_y + w_y]$$

$$-\mathbf{w} = [-w_x, -w_y]$$

$$\alpha \mathbf{w} = [\alpha w_x, \alpha w_y]$$

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## Dot product in coordinates



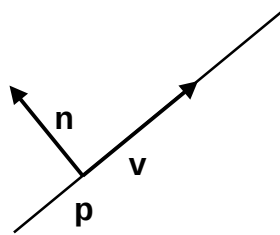
Linear properties become obvious:

$$(v+w, u) = (v, u) + (w, u)$$

$$(av, w) = a(v, w)$$

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## Line equations



parametric equation:

$$q(t) = p + vt$$

$t$  = parameter

$p$  = point on the line

$n$  = unit normal

$v$  = unit vector along

implicit equation:

$$(n, q - p) = 0$$

if we know  $n = [n_x, n_y]$ , then we can take  $v$  to be  $v = [n_y, -n_x]$ , and the other way around.

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## Line equations

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Intersecting two lines:

take one in implicit form:  $(q - p^1, n^1) = 0$

the other in parametric:  $q = p^2 + v^2 t$

If  $q^i = p^2 + v^2 t^i$  is the intersection point, it satisfies both equations.

Plug parametric into implicit, solve for  $t^i$  :

$$(p^2 + v^2 t^i - p^1, n^1) = 0$$

$$\text{If } (v^2, n^1) \neq 0, \text{ then } t^i = -\frac{(p^2 - p^1, n^1)}{(v^2, n^1)}$$

Otherwise, the lines are parallel or coincide.