

Geometry Review

Due date: Wednesday, September 22, before class

In all cases, the direction and perpendicular vectors of the lines and normals to the planes are assumed to have unit length. The problem marked with * is optional; extra credit will be given for it. Providing an answer is insufficient; you have to explain how you have arrived at the answer.

1. Determine which of the following pairs of lines intersect, and find the intersection points; explain your calculations.
 - (a) the line with perpendicular $[-3, 1]$ passing through $[2, 1]$, and the line $x + y = 12$;
 - (b) the line with direction $[1, 2]$ intersecting the y axis at 3 and the line with equation $2x - y + 5 = 0$;
 - (c) the line with direction $[1, 1]$ passing through $[0, 1]$ and the line with normal $[1, -1.0001]$ passing through the origin.
2. Find the expression for the distance from a point $\mathbf{p}_0$ to a line with normal $\mathbf{n}$ passing through the point $\mathbf{p}$.
3. Derive the parametric and implicit equations of the line passing through two given points $\mathbf{p}_1$ and $\mathbf{p}_2$.
4. Derive an equation that has to be satisfied by three points $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$ which are collinear, that is, lie on the same line.
5. Determine when a line with direction $\mathbf{v}$ passing through point $\mathbf{p}$ intersects the ellipse $x^2/a^2 + y^2/b^2 = 1$. In the cases when there is an intersection, find the expressions for intersection points.
6. Derive the implicit equation of the plane passing through points $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$, when such plane is unique. State the condition on $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$ for the plane to be unique.
7. Given two planes with normals $\mathbf{n}_1$ and $\mathbf{n}_2$ passing through points $\mathbf{p}_1$ and $\mathbf{p}_2$ respectively, find the parametric equation for their line of intersection, if any.
8. Find the angle between a line in space with direction $\mathbf{v}$ and the plane with normal $\mathbf{n}$.
9. Find the parametric equation of the line satisfying the following three conditions:
 - (a) it is parallel to a plane p with normal $\mathbf{n}$ passing through a point $\mathbf{p}_1$;
 - (b) it is perpendicular to the line l with direction $\mathbf{v}$ passing through the point $\mathbf{p}_2$;
 - (c) it passes through the intersection point of the line l and the plane p .
10. * Find the distance between two lines $\mathbf{p}_1 + \mathbf{v}_1 t$ and $\mathbf{p}_2 + \mathbf{v}_2 t$ in space.