
Filtering

© 1999, Denis Zorin

Fourier transform

Decomposes into freq. components

$$\mathcal{F}[x](\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t} dt$$

Inverse transform reconstructs the function

$$\mathcal{F}^{-1}[X] = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{i\omega t} d\omega$$

© 1999, Denis Zorin

Examples of FT pairs

FT(delta function) = constant or wave

FT(constant or wave) = delta function

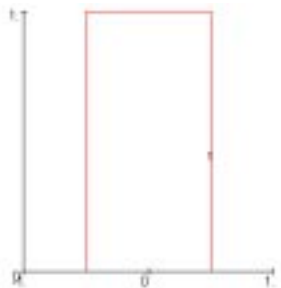
delta function (not quite a function):

for all $x \neq 0$, $\delta(x) = 0$

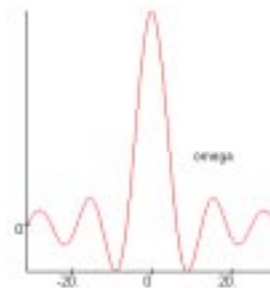
for all ϵ , $\int_{-\epsilon}^{+\epsilon} \delta(x) dx = 1$

© 1999, Denis Zorin

Examples of FT pairs



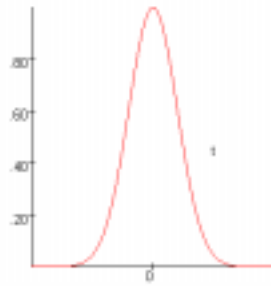
box



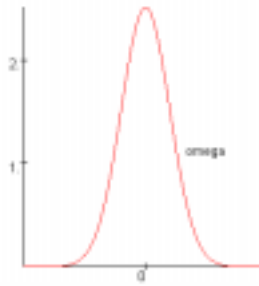
sinc

© 1999, Denis Zorin

Examples of FT pairs



Gaussian

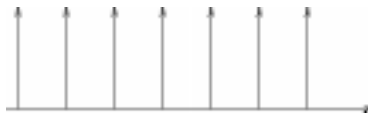


Gaussian

© 1999, Denis Zorin

Comb function

$$comb = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$



FT of a comb is also a comb

© 1999, Denis Zorin

Properties

Name of the property	signals	Fourier transforms
Linearity	$ax(t) + by(t)$	$a\mathcal{F}[x](\omega) + b\mathcal{F}[y](\omega)$
Time shifting (Section 3)	$x(t - t_0)$	$e^{-i\omega t_0} \mathcal{F}[x](\omega)$
Zooming (Section 4.1)	$x(t/a)$	$a\mathcal{F}[x](a\omega)$
Differentiation	$dx(t)/dt$	$i\omega \mathcal{F}[x](\omega)$
Convolution (Section 2.2)	$x(t) * y(t)$	$\mathcal{F}[x](\omega) \mathcal{F}[y](\omega)$

© 1999, Denis Zorin

Shannon theorem

A function can be reconstructed from samples if

- it is bandlimited (Fourier transform is zero for large frequencies)
- sampling frequency is at least twice the max. frequency of the function

© 1999, Denis Zorin

Filtering

solution to aliasing problems: get rid of high frequencies we cannot

reconstruct

In freq. domain: multiply by a box

what does it mean in the spatial domain?

© 1999, Denis Zorin

Convolution

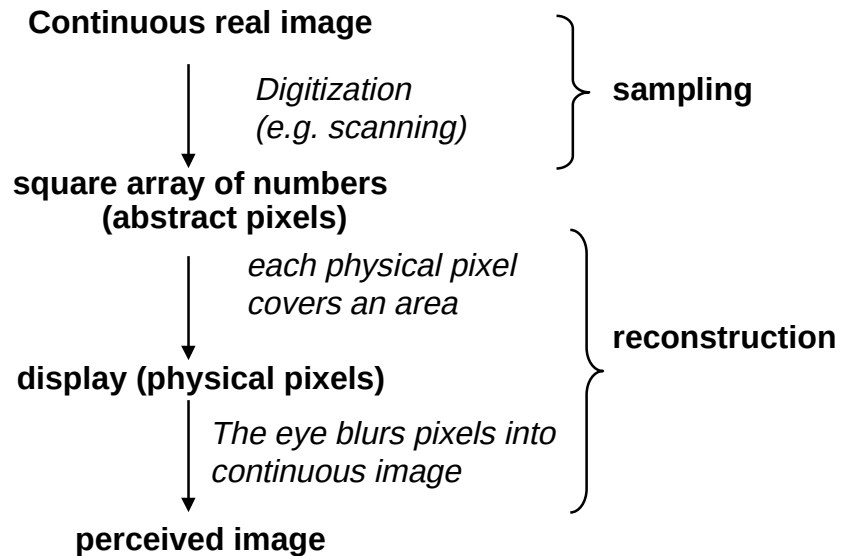
multiplication in the frequency domain is convolution in the spatial domain

$$U(\omega) = X(\omega)Y(\omega)$$

$$u(t) = \int_{s=-\infty}^{+\infty} x(s)y(t-s)ds$$

© 1999, Denis Zorin

Review



© 1999, Denis Zorin

Shrinking



original



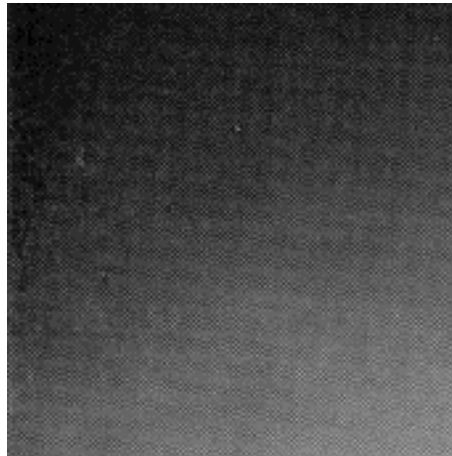
resized, nearest neighbor



resized, 11-point filter

© 1999, Denis Zorin

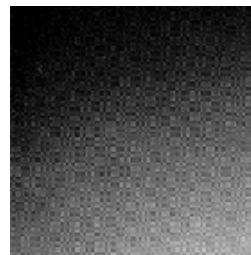
Shrinking



original



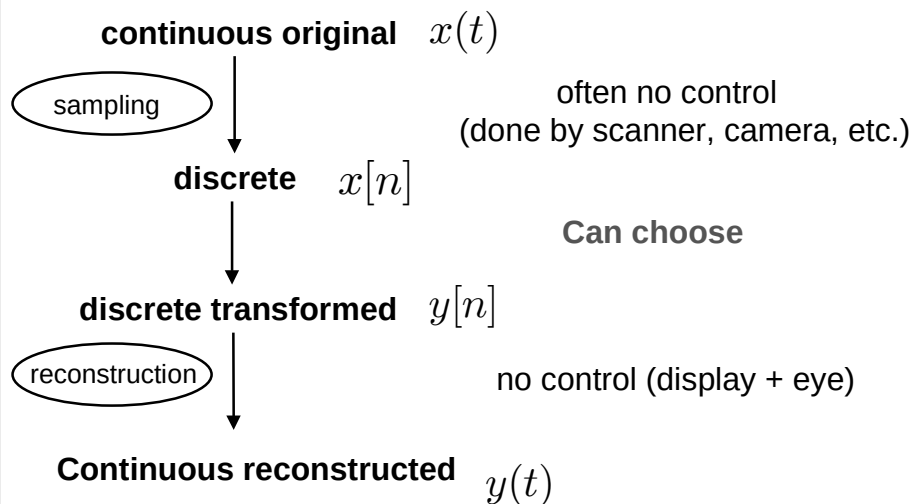
resized, nearest neighbor



resized, 11-point filter

© 1999, Denis Zorin

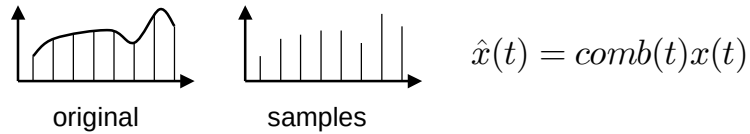
Image transformation



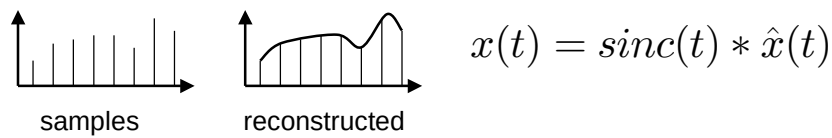
© 1999, Denis Zorin

Sampling and reconstruction

Sampling = multiplication by comb



Reconstruction = convolution with sinc



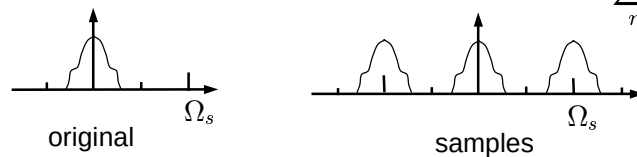
© 1999, Denis Zorin

Sampling and reconstruction

Frequency domain

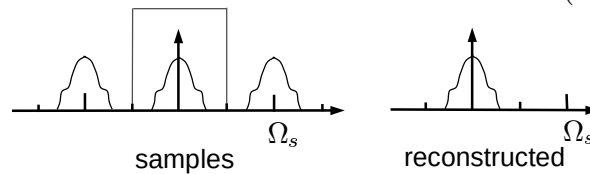
Sampling = convolution with comb
= replication and shift

$$\hat{X}(\omega) = \sum_n X(\omega - n\Omega_s)$$



Reconstruction = multiplication by box (get rid of extra copies)

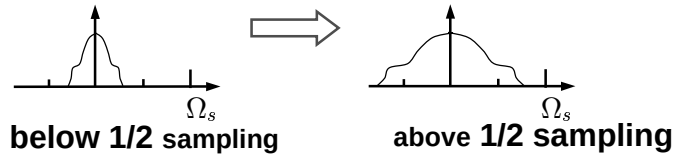
$$X(\omega) = \text{box}(\omega) \hat{X}(\omega)$$



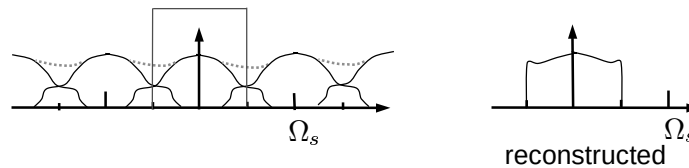
© 1999, Denis Zorin

Shrinking: problem

Shrinking by a factor a
in freq. domain becomes stretching by a



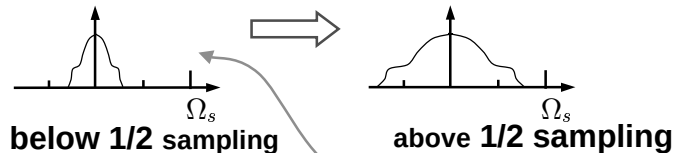
Won't be able to reconstruct correctly
= won't see the expected image



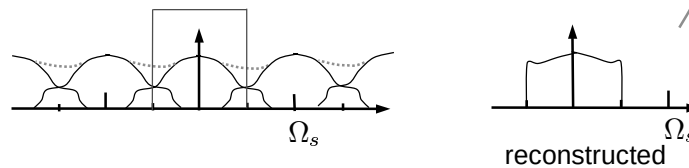
© 1999, Denis Zorin

Shrinking:problem

Shrinking by a factor $a < 1$
in freq. domain becomes stretching by $1/a$



Won't be able to reconstruct correctly
= won't see the expected image



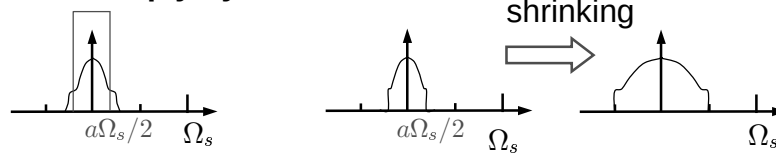
compare
to the original

© 1999, Denis Zorin

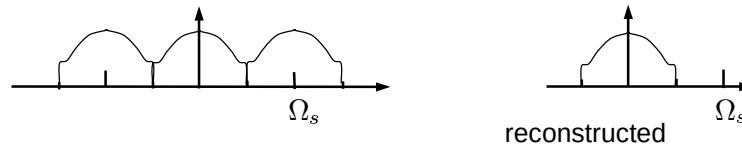
Shrinking: solution

Theoretical solution:

BEFORE shrinking, remove high frequencies,
i.e. multiply by a narrow box



Now, there is no overlap, can reconstruct
(= see the right thing)

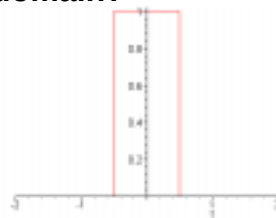


© 1999, Denis Zorin

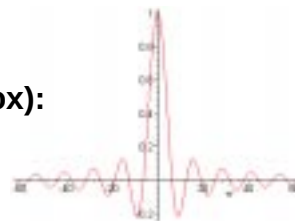
Filters

Theoretical quality criterion: how close to box in
freq. domain?

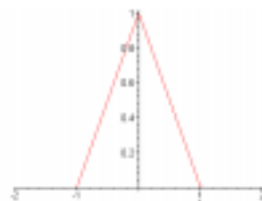
box:



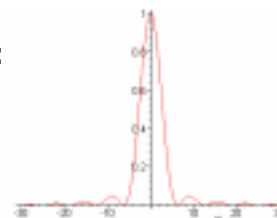
FT(box):



hat:



FT(hat):



© 1999, Denis Zorin

Filters: properties

Other important properties:

symmetric
integrates to 1 (preserve brightness)
short (efficiency)

Generalization to 2D:

just take $h(x)h(y)$;
do all operations twice, once for vertical,
once for horizontal direction

© 1999, Denis Zorin

Filters: boundary

Real signals and images are finite.

What do we do at the boundary?

- everything outside is zero
(may get darker edges)
- pixels outside is the same as on the boundary
- reflect the image across boundaries

© 1999, Denis Zorin

Physics of Light

© 1999, Denis Zorin

Local Illumination model

interaction of light with the surface

Need to know

- how to measure light
- how to describe surface properties
- computer representation

© 1999, Denis Zorin

Properties of light

- spectrum (energy per wavelength)
- polarization
- coherence

Radiometry: physical properties

Photometry: perceptual properties

Visible wavelengths: 380 nm - 770 nm

© 1999, Denis Zorin

Basic Units

Force:

$$\text{Newton} = \text{kg} \cdot \text{m}/\text{sec}^2$$

Energy:

$$\text{joule} = \text{Newton} \cdot \text{m}$$

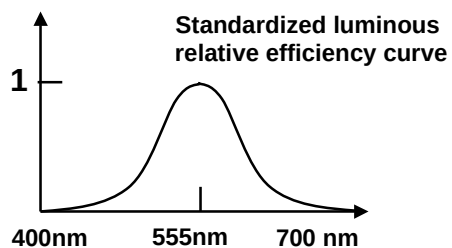
Power:

$$\text{watt} = \text{joule}/\text{sec}$$

To get “standard” eye response, integrate spectrum (energy as function of wavelength) multiplied by relative efficiency.

Luminous energy: talbot;

Luminous power: lumen = talbot/sec



© 1999, Denis Zorin

Photometry and Radiometry

Radiometry units are primary. If the spectrum of light $P(\lambda)$ (measured in watts/nm) is known, then luminous power is computed as

$$684 \int V(\lambda) P(\lambda) d\lambda$$

684 is an arbitrary constant measured in lumens/watt (luminosity at the wavelength 555 nm, yellow-green). If most of the energy of a light source is near 555nm, then to convert from watts to lumens multiply by 684.

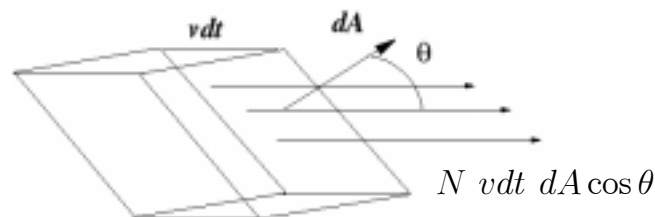
© 1999, Denis Zorin

Flow of light

Assumptions:

light consists out of particles (ignore wave nature)
propagates along straight rays (isotropic medium)

Flow:



N particle density

dA differential area

v particle velocity

© 1999, Denis Zorin

Flux and Flux Density

Flux = particles/unit time; differential flux through a small area:

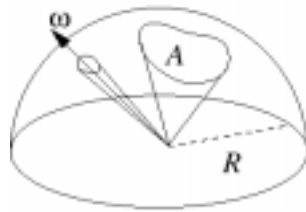
$$d\Phi = Nv \cos \theta dA$$

Flux density = particles/(unit time unit area)

$$\frac{d\Phi}{dA} = Nv \cos \theta$$

© 1999, Denis Zorin

Solid Angles



solid angle spanned by a cone is measured by the area of intersection of the cone with a sphere:

$$\Omega = \frac{A}{R^2}$$

differential solid angle can be assigned a direction. Unit: steradian (full sphere = 4π)

© 1999, Denis Zorin

Measuring light

For any point in space, we can consider directional distribution of photons going through a differential area at this point.

Radiance: energy per unit time, per unit differential area perpendicular to the ray, per unit solid angle in the direction of the ray.

Measured in watts/meter²/steradian

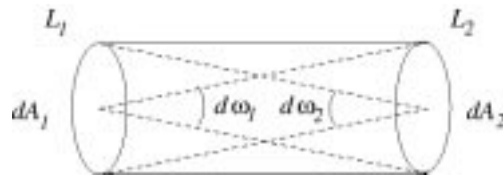
If $\phi(x, \omega) = \frac{dN}{d\Omega}$ is directional distribution of

photons of wavelength λ , going through the area

then radiance is $L(x, \omega, \lambda) = \frac{hc}{\lambda} \phi(x, \omega)$
 energy of a photon

© 1999, Denis Zorin

Constancy of Radiance



radiance is constant along a ray: consider the flow of photons in a thin pencil; the number of photons entering on the right with the direction inside $d\omega_1$ exit through the other side; equating the expressions for entering and exiting diff. flows we get

$$d\Phi_1 = L_1 d\omega_1 dA_1 = L_2 d\omega_2 dA_2 = d\Phi_2$$

$$\text{but } dA_1 d\omega_1 = dA_2 d\omega_2 \text{ so } L_1 = L_2$$

© 1999, Denis Zorin