Scan conversion

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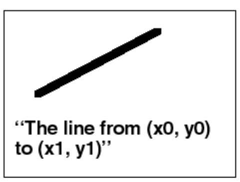
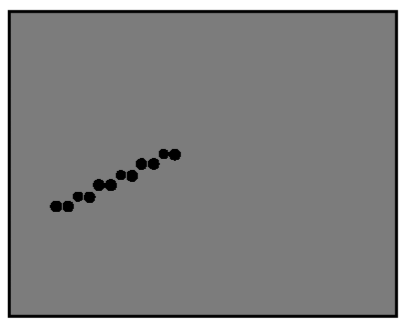
Problem

...to this.

From this...



An Ideal Geometric Primitive



A Bag of Pixels

Drawing on a raster

- Have to represent continuous shapes on a fixed resolution grid
- primitives: lines, polygons, curves
- antialiasing: use changes in intensity to compensate for resolution
- algorithms have to be fast
- algorithms have to be consistent

Primitives

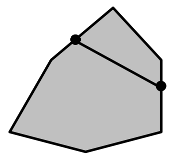
Curves:

	implicit	parametric
general	$f(x,y)=0$	$(x(t),y(t))$
line	$f(x,y)=ax+by-c$	$x(t)=s_x+t(e_x-s_x)$ $y(t)=s_y+t(e_y-s_y)$
circle	$f(x,y)=(x-c_x)^2+(y-c_y)^2-r^2$	$x(t)=c_x+r\cos(2\pi t)$ $y(t)=c_y+r\sin(2\pi t)$

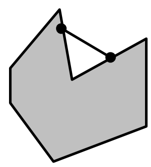
Primitives

Polygons:

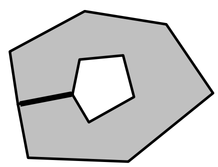
convex



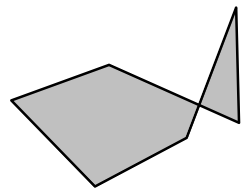
non-convex



with holes



with self-intersections



DDA

- **Take unit steps in *x*, size *m* steps in *y*. Round *y* to the nearest pixel. (Reflect if slope > 1.)**

```
void line (int x0, int y0, int x1, int y1){
    float y = y0;
    float slope = (y1 - y0)/ (float) (x1 - x0);
    int x;
    for(x=x0;x<=x1;x++){
        draw_pixel(x,Round(y));
        y += slope;
    }
}
```

- **This algorithm goes by the grandiose name of *Differential Digital Analyzer* (DDA.)**
- **Problem: Round and floating-point add are expensive (esp. if you're planning to build custom pixel-painting engines!)**

Bresenham algorithm

```
/* this does the right thing (same as DDA) at */
/* a cost of 2 or 3 integer adds per point.   */
/* (assumes sorted endpoints, 0<slope<1)      */

void draw_line(int x0, int y0, int x1, int y1)
   int x,y=y0;dx = 2*(x1-x0),dy = 2*(y1-y0)
   int dydx = dy - dx, d = dy - dx/2;

   for(x=x0;x<=x1;x++){
      draw_pixel(x,y);
      if(d<0) {d+= dy;}
      else    {y+=1; d += dydx;}
   }

}
```

Line $\mathbf{L}$ *from* $[x_0, y_0]$ *to* $[x_1, y_1]$.

$\mathbf{P}_0 = [x_0, y_0]$,

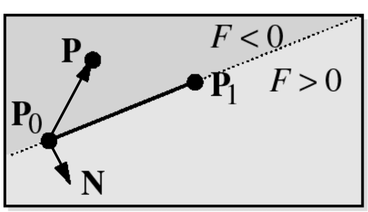
$\mathbf{P}_1 = [x_1, y_1]$.

$dx = x_1 - x_0,\ dy = y_1 - y_0$

$\mathbf{N} = [dy, -dx]$

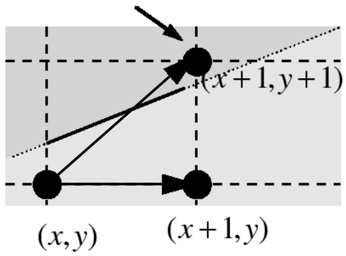
implicit function : $F(\mathbf{P}) = 2\mathbf{N} \cdot (\mathbf{P} - \mathbf{P}_0)$

$F = 0 \rightarrow \mathbf{P}$ *is on* $\mathbf{L}$



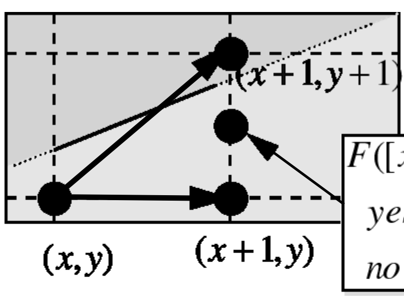
Why the factor of 2? Because we're going to divide by 2 later.

Which pixel is next?



- **Assume 0 < slope < 1, sorted endpoints.**
- **At each step:**
  - **We're at *(x,y)*.**
  - **Should we increment both x and y, or only x?**
  - **Answer: pixel that's closest to the actual line.**
- **We can use the implicit function to decide…**

Using ther implicit equation



$F([x+1, y+1/2]) > 0$?

yes: *increment x and y*

no: *increment x*

NOTE: Blinn's law of sign flips…

Incremental update for F

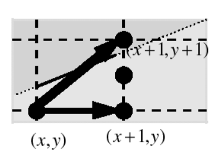
$$F(\mathbf{P}) = 2\mathbf{N} \cdot (\mathbf{P} - \mathbf{P}_0)$$
$$F(\mathbf{P}+\Delta) = 2\mathbf{N} \cdot (\mathbf{P} + \Delta - \mathbf{P}_0)$$
$$= F(\mathbf{P}) + 2\mathbf{N} \cdot \Delta$$

- **Computing $F(\mathbf{P})$ from scratch requires a dot product, but computing a new value given the old one and a position offset only requires an add. (an *integer add,* for us.)**
- **The thing you add in doesn't depend on P so you can precompute and re-use it.**

Bresenham Algorithm

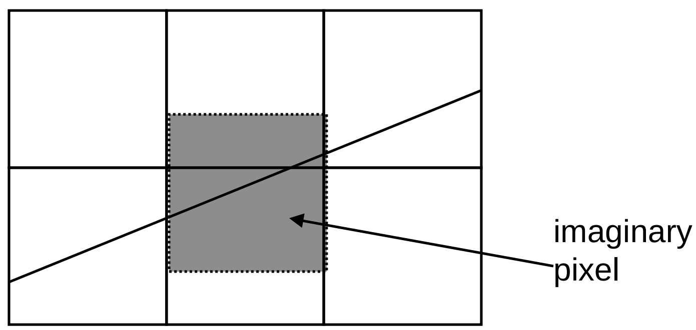
$$F_0 = F(\mathbf{P}_0 + [1, 1/2])$$
$$= F(\mathbf{P}_0) + \mathbf{N} \cdot [2,1]$$
$$F' = F + 2\mathbf{N} \cdot \Delta$$
where
$$\Delta = [1,0] \text{ or } [1,1]$$
$$\mathbf{N} = [dy, -dx]$$
so
$$F' = F + 2dy$$
or
$$F' = F + 2dy - 2dx$$

- **Initialize x,y,F**
- **Eval F one over and 1/2 up.**
- **increment x.**
- **if F>0, increment y, too.**
- **increment F using the appropriate constant (integer) update.**
- **Oh, yeah. Draw the pixel.**
- **Continue to end of line.**



Anti-aliasing

Instead of binary decision, adjust intensity proportoinal to the area:



Anti-aliasing

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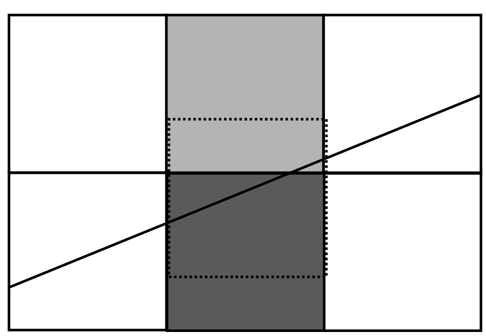
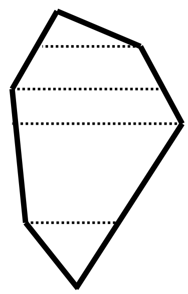


Scan Conversion of Convex Polygons

General idea:

decompose polygon into tiles

scan convert each tile, moving along one edge



Convex Polygons

Scan convert a convex polygon:

```
ScanY( Vertex v[], int nv, int bot )
```

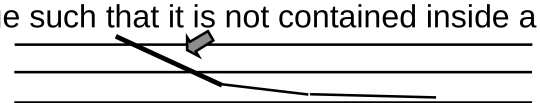
array of vertices in counterclockwise order

array size

number of the vertex with min. y coordinate

1. Find left edge of a tile:

•go around **clockwise,** starting from v[bot], until find an edge such that it is not contained inside a scan line:



2. Similarly, find the right edge of a tile.

3. Scan convert all scan lines going from left to right edges

Convex Polygons

find left boundary edge of a tile

find right boundary edge of a tile

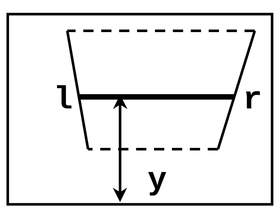
scan convert lines

```
ScanY( Vertex v[], int nv, int bot ){
   int li = bot, ri = li;
   int y = ceil( v[bot].y ), ly = y, ry = y;
   Vertex l, dl, r, dr;
   for( int rem = nv; rem > 0; ){
      while( ly <= y && rem-- > 0 ){
         int i = li; li = mod( li-1, nv ); ly = ceil( v[li].y );
         if( ly > y ) DiffY( v[i], v[li], l, dl, y );
      }
      while( ry <= y && rem-- > 0 ){
         int i = ri; ri = mod( ri+1, n ); ry = ceil( v[ri].y );
         if( ry > y ) DiffY( v[i], v[ri], r, dr, y );
      }
      for( ; y < ly && y < ry; y++ ) {
         ScanX( l, r, y ); l += dl; r += dr; }
   }
}
```


Convex Polygons

Scan conversion of a single scan line:

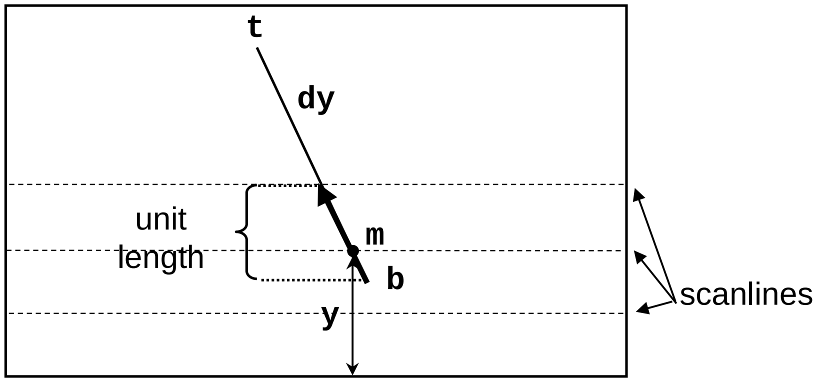
```
ScanX( Vertex& l, Vertex& r, float y ){
   int lx = ceil( l.x ), rx = ceil( r.x );
   if( lx < rx ){
      for( int x = lx; x < rx; x++ ){
         Point( x, y );
      }
   }
}
```



Convex Polygons

Compute increment in y direction and starting/ending point for the first scan

```
DiffY( Vertex& b, Vertex& t, Vertex& m, Vertex& dy, int y ){
   dy = (t - b) / (t.y - b.y); m = b + (y - b.y) * dy;
}
```



Texture mapping for polygons

- map an image to a polygon (e.g. quadrilateral)
- can be done during scan conversion
- can be done using scaling but the scale factor depends on pixel
- to figure out the dependence, need to know the map from the quadrilateral to the image.