Review

Continuous real image

Digitization
(e.g. scanning)

sampling

square array of numbers
(abstract pixels)

each physical pixel
covers an area

reconstruction

display (physical pixels)

The eye blurs pixels into
continuous image

perceived image

Review, Fourier transform

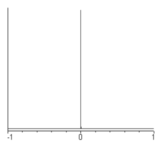
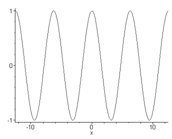
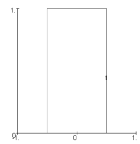
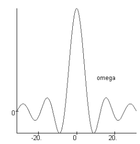
Decomposes into frequency components

$$X(\omega) = \mathcal{F}[x](\omega) = \int_{-\infty}^{\infty} x(t) e^{-i\omega t} dt$$

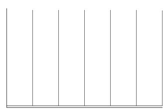
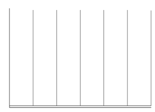
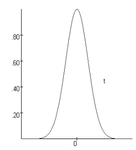
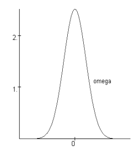
Inverse transform reconstructs the function

$$x(t) = \mathcal{F}^{-1}[X] = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{i\omega t} d\omega$$

Review, FT pairs

delta function **wave** **box** **sinc**

comb **comb** **Gaussian** **Gaussian**

Review, convolution

Convolution with a box = local averaging

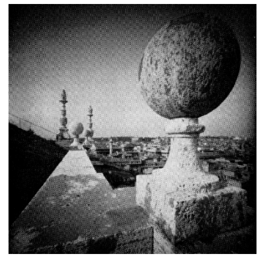
multiplication in the frequency domain
is convolution in the spatial domain:

$$U(\omega) = X(\omega)Y(\omega)$$

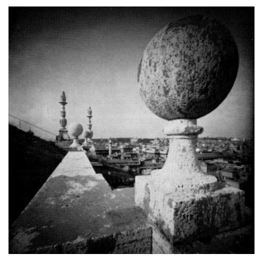
$$u(t) = \int_{s=-\infty}^{+\infty} x(s)y(t-s)ds$$

Shrinking

original

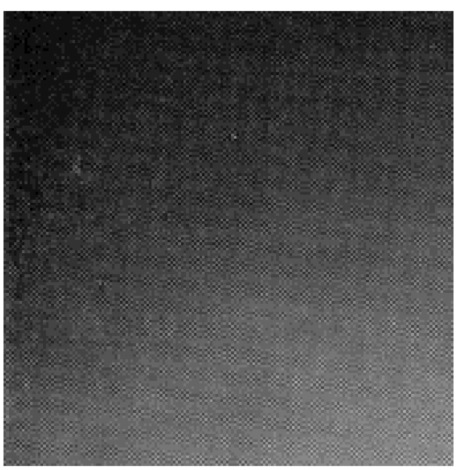


resized, nearest neighbor

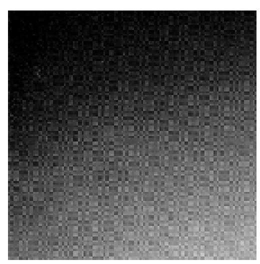


resized, 11-point filter

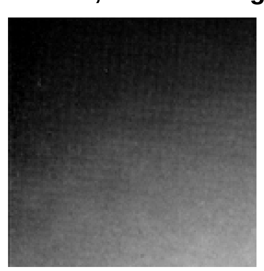
Shrinking



original



resized, nearest neighbor



resized, 11-point filter

Image transformation

continuous original $x(t)$

sampling

often no control
(done by scanner, camera, etc.)

discrete $x[n]$

Can choose

discrete transformed $y[n]$

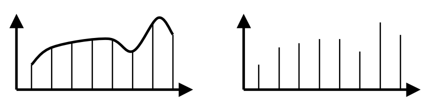
reconstruction

no control (display + eye)

Continuous reconstructed $y(t)$

Sampling and reconstruction

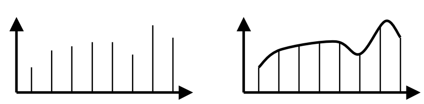
Sampling = multiplication by comb



original samples

$$\hat{x}(t) = comb(t)x(t)$$

Reconstruction = convolution with sinc



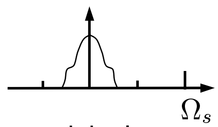
samples reconstructed

$$x(t) = sinc(t) * \hat{x}(t)$$

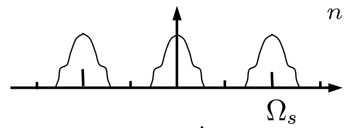
Samping and reconstruction

Frequency domain

Sampling = convolution with comb
= replication and shift $\hat{X}(\omega) = \sum_n X(\omega - n\Omega_s)$

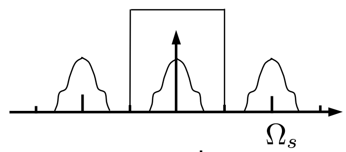


original

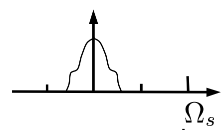


samples

Reconstruction = multiplication by box (get rid of extra copies) $X(\omega) = box(\omega)\hat{X}(\omega)$



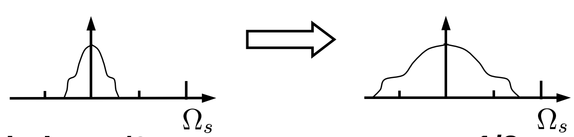
samples



reconstructed

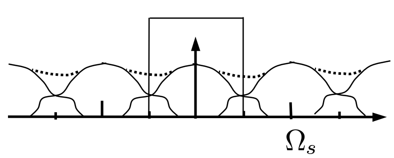
Shrinking: problem

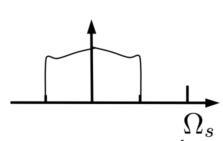
Shrinking by a factor a
in freq. domain becomes stretching by a



below 1/2 sampling **above 1/2 sampling**

Won't be able to reconstruct correctly
= won't see the expected image

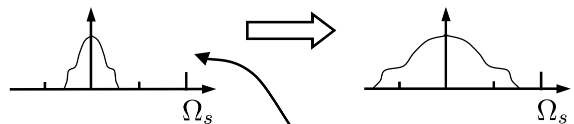




reconstructed

Shrinking:problem

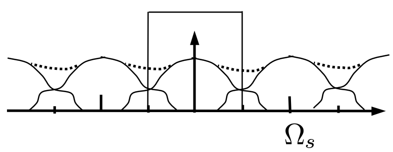
Shrinking by a factor *a < 1*
in freq. domain becomes stretching by *1/a*

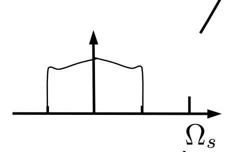


below 1/2 sampling **above 1/2 sampling**

Won’t be able to reconstruct correctly
= won’t see the expected image

compare
to the original

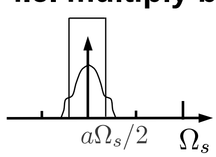




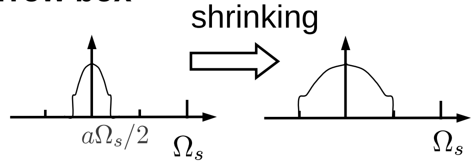
reconstructed

Shrinking: solution

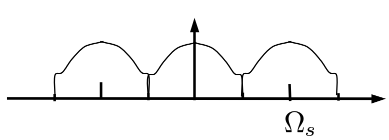
Theoretical solution:
BEFORE shrinking, remove high frequencies,
i.e. multiply by a narrow box

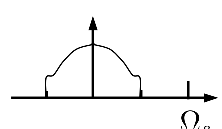


shrinking



Now, there is no overlap, can reconstruct
(= see the right thing)





reconstructed

Shrinking: discrete form

To make it practical, have to reduce to discrete operations

original samples

reconstruct continuous

↓

eliminate high frequency

↓

shrink

↓

sample

Combine into a single discrete operation

resulting samples

Shrinking: discrete form

original samples

reconstruct continuous

↓

eliminate high frequency

↓

shrink

↓

sample

resulting samples

Freq. domain:

$$\left(a\,\mathrm{box}(\omega)\hat{X}(a\omega)\right) * \Omega_s \mathrm{comb}(\omega)$$

spatial domain (after some calculations):

$$y[n] = a \sum_{m=-\infty}^{\infty} x[m]h(T(n-am))$$

$$h = \frac{1}{2\pi} sinc(\frac{\Omega_s t}{2\pi})$$

Filters

Problem: infinite summation!

$$y[n] = a \sum_{m=-\infty}^{\infty} x[m]h(T(n - am))$$

Need to approximate h with finite filters

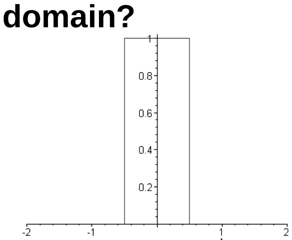
Some choices:

- **box (fast and simple, low quality)**
- **hat (better)**
- **Lanczos, Mitchell, cubic interpolation...**

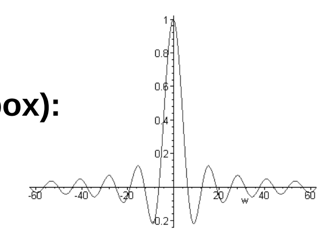
Filters

Theoretical quality criterion: how close to box in freq. domain?

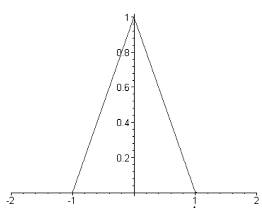
box:



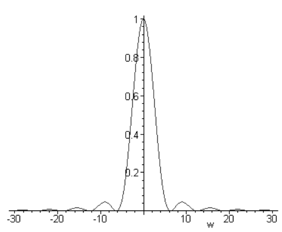
FT(box):



hat:



FT(hat):



Filters: properties

Other important properties:
symmetric
integrates to 1 (preserve brightness)
short (efficiency)

Generalization to 2D:
just take h(x)h(y);
do all operations twice, once for vertical,
once for horizontal direction

Filters: boundary

Real signals and images are finite.

What do we do at the boundary?

- everything outside is zero
  (may get darker edges)
- pixels outside is the same as on the boundary
- reflect the image across boundaries