Image processing

How computer images work?

Continuous real image

Digitization
(e.g. scanning)

square array of numbers
(abstract pixels)

each physical pixel
covers an area

display (physical pixels)

The eye blurs pixels into
continuous image

perceived image

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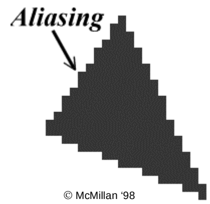
What can go wrong?

Pipeline: sample - process - reconstruct

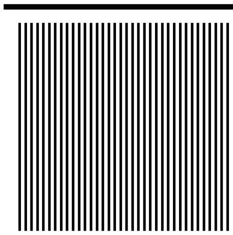
All kinds of artifacts can appear

- **jaggies**
- **alias patterns**
- **moire patterns**
- **temporal aliasing (wheels going wrong way)**

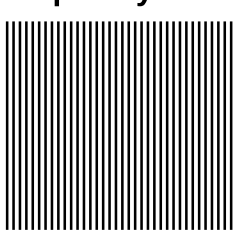


Question: how do we avoid all this?

Aliasing

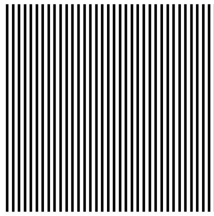


Slightly different frequency

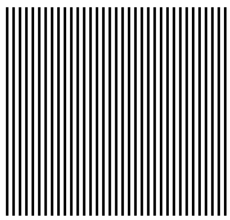


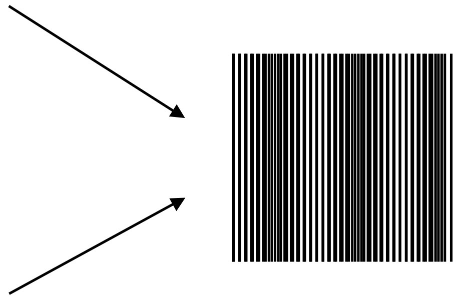
What can go wrong?

Aliasing



Slightly different frequency

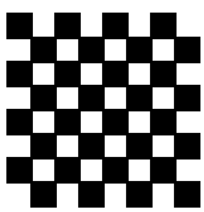




Lower frequency appears

Shrinking

Naïve 1.5x shrinking : drop 2 out of 3

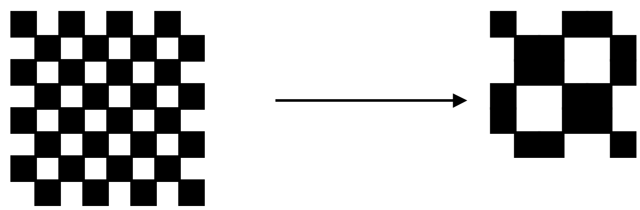


?

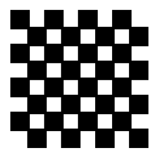
What do we get?

Shrinking

Naïve 1.5x shrinking: drop 2 out of 3



But we want



(impossible)

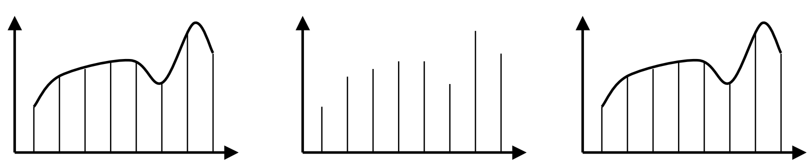
Frequency analysis

The key to fighting aliasing is to

avoid frequencies we cannot represent

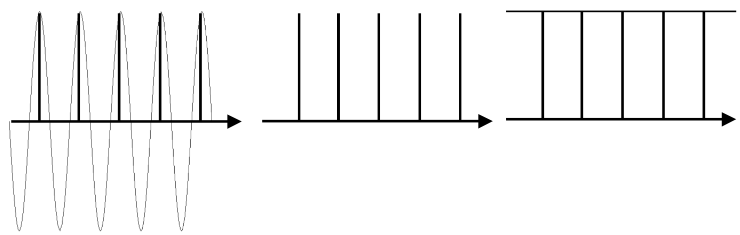
Big question:

When can we reconstruct a continuous signal from samples?



original samples reconstructed

Frequency analysis



original samples reconstructed

Need enough samples, to be more precise, sampling frequency should be twice the frequency of the wave

Frequency analysis

- **decompose everything into waves**
- **mathematically convenient to use complex waves**

$$A\cos(\omega t + \phi)$$

amplitude **frequency** **phase**

$$Ae^{i(\omega t+\phi)} = A\left(\cos(\omega t + \phi) + i\sin(\omega t + \phi)\right)$$

Fourier transform

Decomposes into freq. components

$$\mathcal{F}[x](\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt$$

Inverse transform reconstructs the function

$$\mathcal{F}^{-1}[X] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{i\omega t}d\omega$$

Examples of FT pairs

FT(delta function) = constant or wave

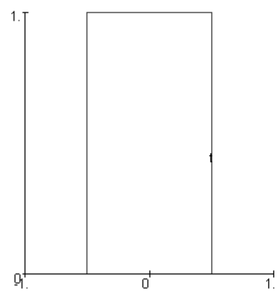
FT(constant or wave) = delta function

delta function (not quite a function):

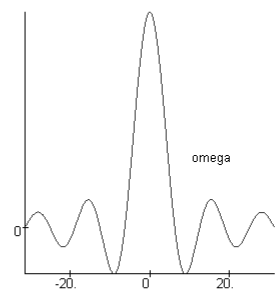
$$\text{for all } x \neq 0\ , \quad \delta(x) = 0$$

$$\text{for all } \epsilon\ , \quad \int_{-\epsilon}^{+\epsilon} \delta(x)dx = 1$$

Examples of FT pairs

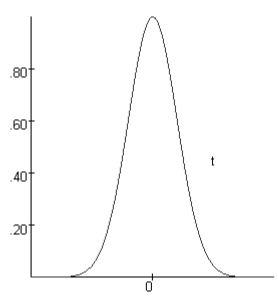


box

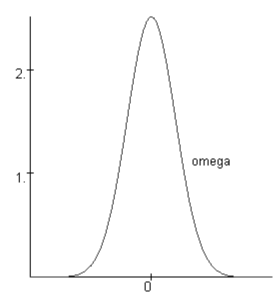


sinc

Examples of FT pairs



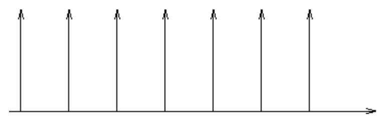
Gaussian



Gaussian

Comb function

$$comb = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$



FT of a comb is also a comb

Properties

Name of the property	signals	Fourier transforms
Linearity	$ax(t) + by(t)$	$a\mathcal{F}[x](\omega) + b\mathcal{F}[y](\omega)$
Time shifting (Section 3)	$x(t - t_0)$	$e^{-i\omega t_0}\mathcal{F}[x](\omega)$
Zooming (Section 4.1)	$x(t/a)$	$a\mathcal{F}[x](a\omega)$
Differentiation	$dx(t)/dt$	$i\omega\mathcal{F}[\omega]$
Convolution (Section 2.2)	$x(t) * y(t)$	$\mathcal{F}[x](\omega)\mathcal{F}[y](\omega)$

Shannon theorem

A function can be reconstructed from samples if

- it is bandlimited (Fourier transform is zero for large frequencies)
- sampling frequency is at least twice the max. frequency of the function

Filtering

solution to aliasing problems: get rid of high frequencies we cannot

reconstruct

In freq. Domain: multiply by a box

what does it mean in the spatial domain?

Convolution

multiplication in the frequency domain is convolution in the spatial domain

$$U(\omega) = X(\omega)Y(\omega)$$

$$u(t) = \int_{s=-\infty}^{+\infty} x(s)y(t-s)ds$$