Image processing

How computer images work?

Continuous real image

↓ *Digitization (e.g. scanning)*

square array of numbers (abstract pixels)

↓ *each physical pixel covers an area*

display (physical pixels)

↓ *The eye blurs pixels into continuous image*

perceived image

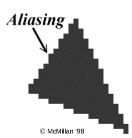
What can go wrong?

Pipeline: sample - process - reconstruct

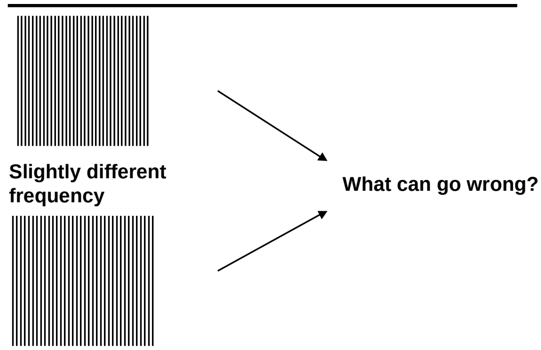
All kinds of artifacts can appear

- **jaggies**
- **alias patterns**
- **moire patterns**
- **temporal aliasing (wheels going wrong way)**

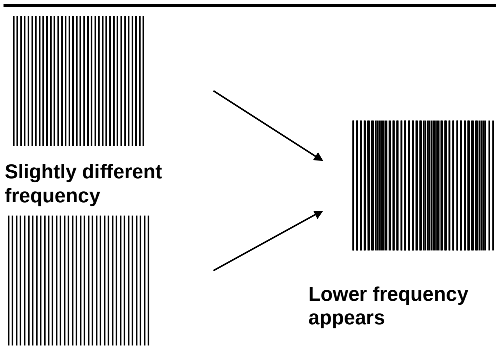
Question: how do we avoid all this?



Aliasing

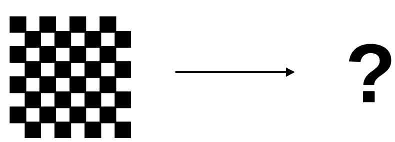


Aliasing



Shrinking

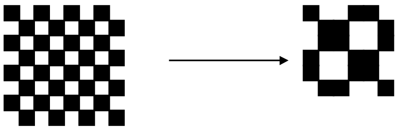
Naïve 1.5x shrinking : drop 2 out of 3



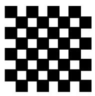
What do we get?

Shrinking

Naïve 1.5x shrinking: drop 2 out of 3



But we want



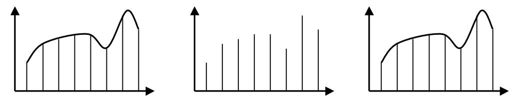
(impossible)

Frequency analysis

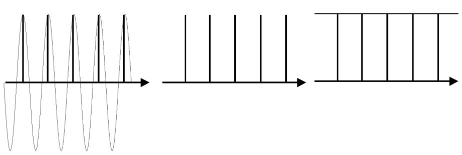
The key to fighting aliasing is to avoid frequencies we cannot represent

Big question:

When can we reconstruct a continuous signal from samples?



Frequency analysis



Need enough samples, to be more precise, sampling frequency should be twice the frequency of the wave

Frequency analysis

- **decompose everything into waves**
- **mathematically convenient to use complex waves**

$$A\cos(\omega t + \phi)$$

amplitude **frequency** **phase**

$$Ae^{i(\omega t+\phi)} = A\left(\cos(\omega t+\phi) + i\sin(\omega t+\phi)\right)$$

Fourier transform

Decomposes into freq. components

$$\mathcal{F}[x](\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt$$

Inverse transform reconstructs the function

$$\mathcal{F}^{-1}[X] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{i\omega t}d\omega$$

Examples of FT pairs

FT(delta function) = constant or wave

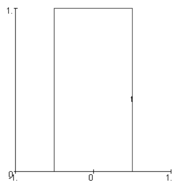
FT(constant or wave) = delta function

delta function (not quite a function):

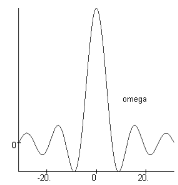
$$\text{for all } x \neq 0\,, \quad \delta(x) = 0$$

$$\text{for all } \epsilon\,, \quad \int_{-\epsilon}^{+\epsilon} \delta(x)dx = 1$$

Examples of FT pairs

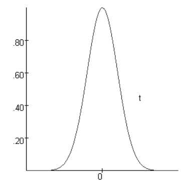


box

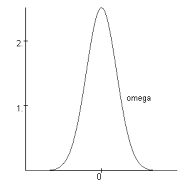


sinc

Examples of FT pairs



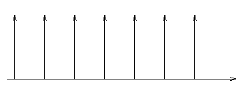
Gaussian



Gaussian

Comb function

$$comb = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$



FT of a comb is also a comb

Properties

Name of the property	signals	Fourier transforms
Linearity	$ax(t) + by(t)$	$a\mathcal{F}[x](\omega) + b\mathcal{F}[y](\omega)$
Time shifting (Section 3)	$x(t - t_0)$	$e^{-i\omega t_0}\mathcal{F}[x](\omega)$
Zooming (Section 4.1)	$x(t/a)$	$a\mathcal{F}[x](a\omega)$
Differentiation	$dx(t)/dt$	$i\omega\mathcal{F}[\omega]$
Convolution (Section 2.2)	$x(t) * y(t)$	$\mathcal{F}[x](\omega)\mathcal{F}[y](\omega)$

Shannon theorem

A function can be reconstructed from samples if

- **it is bandlimited (Fourier transform is zero for large frequencies)**
- **sampling frequency is at least twice the max. frequency of the function**

Filtering

solution to aliasing problems: get rid of high frequencies we cannot

reconstruct

In freq. Domain: multiply by a box

what does it mean in the spatial domain?

Convolution

multiplication in the frequency domain is convolution in the spatial domain

$$U(\omega) = X(\omega)Y(\omega)$$

$$u(t) = \int_{s=-\infty}^{+\infty} x(s)y(t-s)ds$$