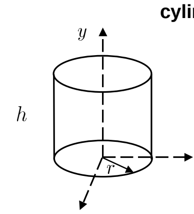
More on Ray-Surface Intersections

Ray-Cylinder Intersections



cylinder of height h, radius r

equations for the side surface:

$x^2 + z^2 = r^2$ (infinite cylinder)

$0 \le y \le h$

equations for the caps:

top: $y = h \quad x^2 + z^2 \le r^2$

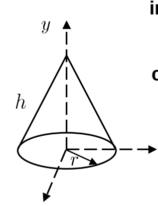
bottom: $y = 0 \quad x^2 + z^2 \le r^2$

Ray: $$p_0 + vt = \begin{bmatrix} p_{0x} + v_x t \\ p_{0y} + v_y t \\ p_{0z} + v_z t \end{bmatrix}$$

Ray-Cylinder Intersections

1. Intersect with the side surface:
   plug in the ray equation into the infinite cylinder equation $(p_{0x} + v_x t)^2 + (p_{0z} + v_z t)^2 = r^2$
   solve for t, (possibly two solutions); choose the positive solution(s) that satisfy $0 \le y \le h$, if any.
2. Intersect with the caps:
   intersect with the planes, find t, check if it is positive; then check if it satisfies $x^2 + z^2 \le r^2$
3. From the set of all intersection points (with the side and caps) if any, choose the one with minimal t.

Ray-Cone Intersection



infinite cone with apex at zero:

$x^2 + z^2 = k^2 y^2$

cone with apex on y axis at h

$x^2 + y^2 = \frac{r^2}{h^2}(y - h)^2$

$0 \le y \le h$

Intersections are computed similar to the case of the cylinder: intersect with the side, intersect with the cap, choose the closest intersection.

Transformed Objects

To intersect with an object given by equation

, $f(p) = 0$ where $p = [x, y, z]$ transformed by a transformation T, given by a 4x4 matrix

with the ray $p_0 + vt$

transform the ray to the coordinate system of the object by applying the inverse transform:

$$T^{-1} p_0 + T^{-1} vt$$

remember that p_0 is a point (last homogeneous coordinate 1), and v is a vector (last coordinate 0).

Lighting

Lighting

Geometric model → Illumination Calculations → Display

Lighting model:
- how light propagates in the media
- how light interacts with objects
  - (local illumination model)
- light representation

Local Illumination model

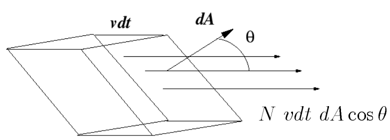
interaction of light with the surface

Need to know
- how to measure light
- how to describe surface properties
- computer representation

Flow of light

Assumptions:
- light consists out of particles (ignore wave nature)
- propagates along straight rays (isotropic medium)

Flow:



N particle density

dA differential area

v particle velocity

Flux and Flux Density

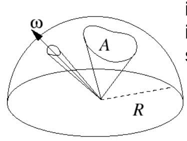
Flux = particles/unit time; differential flux through a small area:

$$d\Phi = Nv\cos\theta dA$$

Flux density = particles/(unit time unit area)

$$\frac{d\Phi}{dA} = Nv\cos\theta$$

Solid Angles



solid angle spanned by a cone is measured by the area of intersection of the cone with a sphere:

$$\Omega = \frac{A}{R^2}$$

differential solid angle can be assigned a direction. Unit: steradian (full sphere = 4π)

Measuring light

For any point in space, we can consider directional distribution of photons going through a differential area at this point.

Radiance: energy per unit time, per unit differential area perpendicular to the ray, per unit solid angle in the direction of the ray.
Measured in watts/meter^2 /steradian

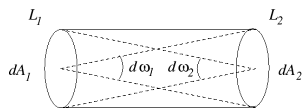
If $\phi(x,\omega) = \frac{dN}{d\Omega}$ is directional distribution of

photons of wavelength λ, going through the area

then radiance is $L(x,\omega,\lambda) = \frac{hc}{\lambda}\phi(x,\omega)$

energy of a photon

Constancy of Radiance



radiance is constant along a ray: consider the flow of photons in a a thin pencil; the number of photons entering onthe right with the direction inside $d\omega_1$ exit through the other side; equating the expressions for entering and exiting diff. flows we get

$$d\Phi_1 = L_1\ d\omega_1 dA_1 = L_2\ d\omega_2 dA_2 = d\Phi_2$$

but $dA_1 d\omega_1 = dA_2 d\omega_2$ **so** $L_1 = L_2$

Wavelength

In natural light all wavelength are present.

We cannot represent all of them.

Solution: use perception: light with arbitrary spectrum produces in a perceptual response equivalent to one produced by a linear combination of 3 primary wavelengths (red, green,blue).In In most cases, three numbers are ok.

Reflections from a surface

Types of reflection

- **ideal specular**
- **ideal diffuse**
- **specular**