Eigenstructure of the Doo-Sabin scheme

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In this worksheet we compute the eigenvalues and eigenvectors of the subdivision matrix of the Doo-Sabin scheme, partially reproducing the analysis of Doo and Sabin, and Peters and Reif.

Utilities

Doo-Sabin subdivision matrix, eigenvalues, eigenvectors

```
> assume( c, real); additionally( c >= -1 ); additionally( c  <= 1);
> DooSabin := matrix( [[ (1/4)*d+1/2 + (1/4)*c, 0,0,0],
                      [    9/16 + (3/16)*omega, 3/16 ,omega/16,0],
                      [    9/16 + (3/16)*conjugate(omega), conjugate(omega)/16, 3/16,0],
                      [    9/16,  3/16, 3/16, 1/16]
                      ]);
```

$$DooSabin := \begin{bmatrix} \frac{1}{4}d + \frac{1}{2} + \frac{1}{4}c\sim & 0 & 0 & 0 \\ \frac{9}{16} + \frac{3}{16}\omega & \frac{3}{16} & \frac{1}{16}\omega & 0 \\ \frac{9}{16} + \frac{3}{16}\overline{\omega} & \frac{1}{16}\overline{\omega} & \frac{3}{16} & 0 \\ \frac{9}{16} & \frac{3}{16} & \frac{3}{16} & \frac{1}{16} \end{bmatrix}$$

```
> DSvar := { omega = c + I*s};
```

$$DSvar := \{\omega = c\sim + I\,s\}$$

```
> evalues := subs( s^2 = 1-c^2, [eigenvals(map( evalc, subs( { d = 0, op(DSvar)}, evalm(DooSabin))))]);
```

$$evalues := \left[\frac{1}{2} + \frac{1}{4}c\sim, \frac{1}{16}, \frac{1}{4}, \frac{1}{8}\right]$$

```
> EV := 1/2 + 1/4*c;
```

$$EV := \frac{1}{2} + \frac{1}{4}c\sim$$

```
> DSZero := map( evalc, subs( { d = 1, c = 1, omega = 1}, evalm(DooSabin))):
> eigenvals( DSZero);
```

$$1, \frac{1}{16}, \frac{1}{8}, \frac{1}{4}$$

```
> evects := subs( s^2 = 1-c^2, [eigenvects(map(evalc,subs( { d = 0, op(DSvar)}, evalm(DooSabin))))]);
```

$$evects := \left[\left[\frac{1}{4}, 1, \{[0, 1, -I\,s + c\sim, 1 - 2\,I\,s + I\,s^3 + I\,s\,c\sim^2 + 2\,c\sim - c\sim\,(1 - c\sim^2) - c\sim^3]\}\right], \left[\frac{1}{8}, 1, \{[0, 1, I\,s - c\sim, 3 + 2\,I\,s + I\,s^3 + I\,s\,c\sim^2 - 2\,c\sim - c\sim\,(1 - c\sim^2) - c\sim^3]\}\right],$$
$$\left[\frac{1}{2} + \frac{1}{4}c\sim, 1, \{\left[1, -3\,\frac{16 + 4\,c\sim^2 + 4\,I\,s\,c\sim + 8\,I\,s + 20\,c\sim}{-16\,c\sim^2 - 40\,c\sim - 24}, -3\,\frac{4\,c\sim^2 + 20\,c\sim + 16 - 4\,I\,s\,c\sim - 8\,I\,s}{-16\,c\sim^2 - 40\,c\sim - 24}, -9\,\frac{24\,c\sim^2 + 80\,c\sim + 56}{(-16\,c\sim^2 - 40\,c\sim - 24)\,(7 + 4\,c\sim)}\right]\}\right], \left[\frac{1}{16}, 1, \{[0, 0, 0, 1]\}\right]\right]$$

```
> for i from 1 to vectdim(evects) do
    if op(1,op(i, evects)) = EV then v := op(1, op(3, op(i, evects))); fi;
  od;
> DSEigenvector := evalm( map( factor, map(simplify, map(evalc, eval(v)))));
```

$$DSEigenvector := \left[1, \frac{3}{2}\,\frac{I\,(-I\,c\sim^2 - 5\,I\,c\sim + c\sim\,s + 2\,s - 4\,I)}{(2\,c\sim + 3)\,(c\sim + 1)}, -\frac{3}{2}\,\frac{I\,(I\,c\sim^2 + 5\,I\,c\sim + c\sim\,s + 2\,s + 4\,I)}{(2\,c\sim + 3)\,(c\sim + 1)}, 9\,\frac{3\,c\sim + 7}{(7 + 4\,c\sim)\,(2\,c\sim + 3)}\right]$$

Generate code